# Numerical Model of Fish Exploitation - structure and application

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Abstract. Fisheries simulation models are tools used for forecasting the effects of exploitation and determining the directions of managing fisheries resources. The Numerical Model of Fish Exploitation (NMFE) and its capabilities were tested on a population of common bream, Abramis brama (L.) in a dam reservoir that is exploited by commercial and recreational fisheries. Based on the designated population parameters of  $N_0$ ,  $F_{ij}$ ,  $M_i$ , and  $e_i$  and the size and structure of the common bream population in the reservoir, the model was used to examine hypothetical simulation variants of changes in fishing intensity  $E_1$  with nets and rods, changes in fishing intensity based on actual fishing effort with nets, changes in natural mortality, changes in the size of fish caught, and the impact of this on the size of the resources. Initial catches with nets and rods were calculated. Increasing fishing effort did not translate proportionally to increased catches, and the function was curvilinear. The results of simulations that reduced the intensity of fishing with nets and decreased catch sizes concurred with data from actual catches. Simulations of changes in natural mortality had various effects on the size of catches. Reducing parameter M did not impact the level of catches, while increasing parameter M reduced the size of catches significantly.

**Keywords:** common bream, simulation model, catches, nets and rods, natural and fishing mortality

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### Introduction

The basis for the rational management of fishery resources in open waters is to maintain exploitation at levels at which catching biomass surplus permits maintaining populations in good biological condition. Therefore, depending on current needs resulting from maintaining fish stocks, rational fisheries management is understood as introducing changes in the exploitation parameters of fishing intensity, catch size, structure of the fished population, and the fishing system. Simulation fishing models are useful for studying these phenomena as they permit tracking the dynamics of fish populations under the influence of fishing and anticipating the effects of exploitation (Schaefer 1954, Beverton and Holt 1957, Walters 1969, Hightower and Gilbert 1984, Kompowski and Horbowy 1990 - description of the Shaefer, Pella-Tomlinson, Fox, Gulland, and Pope models).

Single-species models refer to the exploitation of just one species and population dynamics are considered from the perspective of exploitation (Tyurin 1962). Simulations conducted with these models refer to changes in population parameters: natural and fishing mortality, exploitation age, and also initial abundance and recruitment (Schaefer 1957, Krishnan and Qasim 1968, Cushing 1971, Biró 1978, Bandura and Shybaev 1986, Kazanskiy et al. 1986, Shybaev 1986, Powers 2014). Multi-species

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models are more complicated. Stock size and dynamics are considered not only depending on exploitation but also on the relationships among species, e.g., predator and prey, and the impact of the environment (Mandecki 1976, Kazanskiy 1977, Majkowski 1977, Kazanskiy and Koval 1980, Kompowski and Horbowy 1990, Kinzey and Punt 2009). These models examine relationships of fish feeding levels and their impact on these populations when catches are conducted simultaneously. Applying simulation models requires gathering biological data about the fished population, the magnitude of exploitation, and the environment. Therefore, logistic approaches are used that allow, on the one hand, to adapt to the type and scope of data possessed, and, on the other hand,

 Table 1

 Basic symbols and parameters and equations in the NMFE

to the reduce costs and labor intensity of the analyses (Kompowski and Horbowy 1990).

The fisheries productivity of inland waters is variable (Miroshnichenko et al. 1986, Helminen et al. 1993). Fish catches reach levels of approximately 5 kg ha<sup>-1</sup> in rivers and approximately 9-17 kg ha<sup>-1</sup> in lakes and dam reservoirs in Poland (Bieniarz et al. 1990, Epler et al. 2005, Falkowski 2008, Wołos et al. 2008), and can even reach as much as to 12-68 kg ha<sup>-1</sup> (Negonovskaya 1986). The estimated fish biomass in inland waters ranges from 90 to 1300 kg ha<sup>-1</sup> (Epler and Bieniarz 1977, Mastyński 1984, Jelonek and Amirowicz 1987a, 1987b, Horpilla and Peltonen 1994, Mastyński and Klimaszyk 1994, Łysak and Ligaszewski 1998, Wiśniewolski 2002, Prchalová et al. 2009). The abundance of fish in

Symbol	Description
В	- frequency of fish caught in units (individual fish); <i>B</i> 1, <i>B</i> 2, <i>B</i> 3, …, refer to subsequent generations from empiric processing B in simulations
С	– mean fish weight distribution in length classes
U	– frequency of fish in length classes
D	– frequency of fish according to type
$e_i$	– fecundity of a single female
fi	– share of mature females
Ε	<ul> <li>coefficient of changes in fishing intensity</li> </ul>
F	- fishing mortality; F1, F2, F3,, refer to subsequent generations of empiric variable processing in simulations
G	– price per 1 kilogram of fish
i-nth	– rows – matrix of the period of index (maximum age of fish <i>t</i> periods), the equivalent is the rows of the matrix where <i>i</i> = 1,2,3n
j-nth	- columns – the index that refers to any quality linked with the exploitation of a population, e.g., fishing gear, fishing region, fishing vessel, the part of the stock exploited, the part of the population that is not fishes, changes in the scope of fishing, scope of protection, etc.; two matrices of the fishing system in the Zegrzyńskie Reservoir $ j nets $ , $ .j = rods $
k	– index of types, the columns are widened in the matrices
M	– natural mortality
Ν	– population abundance: $N1$ , $N2$ , $N3$ ,, refer to subsequent generations of empiric variable processing in simulations; in the model the first period $i = 1$ refers to the initial abundance of fish, thus the number of eggs or the reproductive potential of population $e$ , thus $N_i = e$
S	– catches in units (individual fish)
t	– annual time period, year of life
Y	– fish catches in kilograms
w	– mean catch weight
Z	- value of catches in PLN



Figure 1. NMFE structure; a) preparatory programs PROM0, PROM1, and PROM2 for calculating fish abundance and population parameters, b) input data for subsequent simulation options.

inland waters creates possibilities to exploit these resources through commercial and recreational catches. Applying various catch systems has an impact on fisheries resources, which is why their exploitation requires finding methods to optimize the conditions of these catches. The aim of this paper was to present the possibilities of applying the Numerical Model of Fish Exploitation (NMFE) to forecast the dynamics of fish populations based on the example of common bream *Abramis brama* (L.) in Zegrzyńskie Reservoir.

## Materials and methods

Table 1 presents an alphabetical list of all the symbols of the matrix and algorithm parameters used in the NMFE structure. The structure of the model was a program with two parts (Figs. 1a-b): NMFE–PROM – which had three peripheral programs–PROM0, PROM1, and PROM2–used for:

<u>PROM0</u> – calculating and totaling fish catches in units  $S_{ij}$ ; the size of the matrices were declared

according to *i*-*n*th lines and *j*-*n*th columns and calculations of the sum of total catches in units  $\sum_{i} S_{ij} = S_{ijj}$ 

<u>PROM1</u> – determining initial number of fish  $N_i$ according to:  $N_i = (N_{i+1} + S_{ij} + S_{i\cdot j}) / (1 - M_i)$ 

In the model the following were declared: number and -nth periods, number of fish outside matrix  $N_{z+1...}$ , the matrices entered were  $S_{ij}$ ,  $M_i$ ,  $f_i$ ,  $e_i$ . The annual parameter of Mi was taken from previous calculations (Nabiałek 1995). Data was collected throughout a calendar year taking into consideration three growth periods: summer (June-September VI-IX), fall-winter (October-February X-II), and spring (March-May III-V), and seasonal natural mortality was determined for common bream in Zegrzyńskie Reservoir according to:  $M_i = 1 - \sqrt[3]{1 - M_i}$ . The program permitted constructing a virtual population and fish generations from birth, the so-called initial value of  $N_i$ , to the death of the last individual.

<u>PROM2</u> – determined the value of  $B_{ij}$  according to  $B_{ij} = S_{ij}/N_i$ . Matrix  $B_{ij}$  had the same format and dimension as matrix  $S_{ij}$ , but it did carry with it information about the share of fish.

Three peripheral PROM program parameters  $N_i$ ,  $M_i$ , and  $B_{ij}$  were determined, and they were introduced in the second part of the model with four simulation options, as follows:

<u>Option 1</u> referred to simulating current common bream exploitation. The basic declaration was the number of years of life *t*, matrix format and size according to *i*-*n*th periods and *j*-*n*th columns, initial values  $N_0$ , and matrix  $B_{ij}$  and weight distribution  $C_{ij}$ . Catches were calculated according to procedures included in the NMFE:

 $S_{ij} = N_i \cdot B_{ij}$  catches in individuals  $Y_{ij} = N_i \cdot B_i \cdot C_{ij}$  catches in kilograms and sorted by type *k*-*nth* type

$$\begin{split} S_{ij|k|} &= N_i \cdot B_{ij} \cdot D_{ij|k|} \text{ units} \\ Y_{ij|k|} &= N_i \cdot B_{ij} \cdot D_{ij|k|} \cdot U_{ij|k|} \text{ kilograms} \\ \text{ and the monetary value of this catch, e.g., in PLN} \\ Z_{ij} \text{ according to: } Z_{ii|k|} &= Y_{ii|k|} \cdot G_k . \end{split}$$

<u>Option 2</u> referred to simulating changes in fish catches by changing index of fishing intensity *E*. In addition to the basic data  $N_i$ ,  $B_{ij}$ ,  $F_{ij}$  distribution of weight  $C_{ij}$  and shares  $U_{ij}$ , natural mortality matirx  $M_i$ , and the index of fishing intensity matrix  $E_{ij}$  were introduced. Where  $E_{ij}$  equalled 1, it referred to initial exploitation (for the 1997-2001 period), while values lower or higher than 1 referred to lesser or greater fishing intensity, respectively. The result of the simulation after introducing the new value of  $E_{ij}$  was to obtain new catches  $S_{ij}$ ,  $Y_{ij}$  and the new population parameter values of  $N_i$ ,  $B_{ij}$ ,  $F_{ij}$  according to:

$$N_i = N_{i-1}(1 - M_i) - \sum_j B_{(i-1)j}$$

Determining fishing mortality  $F_{ij}$  proceeded according to  $F_{ij} = B_{ij} / (N_i(1-M_i))$ , and its new values after changing intensity were according to  $F_{1ij} = F_{ij} \cdot E_{ij}$ 

Determining new matrix  $B1_{ij}$  proceeded in the model according to  $B1_{ij} = N1_i(1-M_i)F1_{ij}$ , where  $N1_i$  referred to the new fish abundance determined according to  $N1_i = N1_{i-1}(1 - M_{i-1})(1 - \sum_j F1_{(i-1)j})$ 

Calculating new fish catches in units or kilograms after changing catch intensity proceeded in the model according to the procedure:

$$S1_{ij} = N_o \cdot B1_{ij}$$
$$Y1_{ij} = N_o \cdot B1_i \cdot C_{ij}$$

The format and size of the basic matrices  $B_{ij}$ ,  $F_{ij}$ ,  $E_{ij}$ , just as the new ones determined, were the same. The monetary value of catches proceeded in accordance with the procedure presented above.

Option 3 referred to simulating changes in exploitation with nets by decreasing the size limit. The initial data were the parameters determined previously of  $N_i$  and  $M_i$ . Data on the weight distribution  $W_i$  of fish in year of life t where i referred to the subsequent period (season) was taken from back calculations. New values of  $B_i$ 1 were generated for this purpose based on the empirical structure of the fish catches made with seines. It was hypothesized that the strongest catches would be those of younger year classes of fish in the second (1+), third (2+), and fourth (3+) years of life. The simulation gave new

Table 2	
Population of common bream at constant catch intensity $E$ with nets a	and rods

Period /Age	Fish	Mortality	Commercial gill-net catches		es	Recreational catches			
<u></u>	abundance	M	c.	V	E	S	V	F	
$\frac{A_{ij}}{0+i}$	<u>Ni</u> 2024840348	01 3	3 <sub>ij</sub>	1 ij•	<u>г</u> ij.	<u> </u>	1 i•j	rij	
0+1	255428112	91.5 01 3							
0+2	22306626	91.5 01 3							
1+4	1948045	41.5				261	22	0.02	
1+=	1138963	41.5				201	22	0.02	
1+6	666070	41.5							
2+7	389520	29.5	175	48	0.1	1042	187	0.4	
2+8	273290	29.5	88	18	0.05	1012	107	0.1	
2+9 2+9	192508	29.5	127	30	0.1	53	6	0.04	
$3^{\pm}10$	135486	18.1	1617	564	1.5	1250	302	1.1	
$3+_{11}$	108139	18.1	503	163	0.6				
$3+_{12}$	88097	18.1	593	278	0.8	53	17	0.1	
$4+_{13}$	71533	11.2	3716	1984	5.8	3907	1947	6.2	
$4+_{14}$	55892	11.2	1575	888	3.2				
4 + 15	48052	11.2	1381	890	3.2	133	60	0.3	
5 + 16	41151	9.1	3410	2416	9.1	4480	2992	12.0	
$5+_{17}$	29498	9.1	2012	1400	7.5				
$5+_{18}$	24789	9.1	1161	932	5.2	80	58	0.4	
$6+_{19}$	21281	9.1	2098	1927	10.9	3022	2550	15.6	
6 + 20	14215	9.1	1859	1738	14.4	114	153	0.9	
$6 + _{21}$	10943	9.1	593	546	6.0	27	26	0.3	
$7 + _{22}$	9323	9.1	1137	1221	13.4	1146	1004	13.5	
7 + 23	6188	9.1	984	1194	17.5				
7 + 24	4638	9.1	212	223	5.0	27	70	0.6	
8 + 25	3975	9.1	787	1047	21.8	573	667	15.9	
$8 + _{26}$	2252	9.1	481	606	23.5				
8 + 27	1565	9.1	34	34	2.4				
$9 + _{28}$	1388	9.1	437	634	34.7	104	160	8.3	
$9 + _{29}$	719	9.1	87	118	13.4				
$9 + _{30}$	566	9.1	17	24	3.3				
$10 + _{31}$	497	7.2	262	484	56.8				
10 + 32	200	7.2	22	55	11.8				
10 + 33	163	7.2							
$11 + _{34}$	152	7.2				52	68	37.0	
11 + 35	89	7.2	22	29	26.6				
$11 + _{36}$	60	7.2							
12 + 37	56	7.2				52	120	100.0	
$12 + _{38}$	0	7.2							
12 + 39	0	7.2							

catches  $S1_i$  and  $Y1_i$ . Changes in reproductive potential as an effect of increased fishing pressure on young fish were calculated with the peripheral PROM1 program, and new values for reproductive potential *e*1 were obtained. For the nth simulation, the record of changes in potential was  $N_0 \neq e$ 1.

<u>Option 4</u> referred to simulating changes in exploitation with simultaneous interference in fishing

Table 3

Common bream population at a constant catch intensity: fish abundance in the stock, natural mortality M, caught Sj, and those remaining  $S_{AV}$  in the reservoir

Age group	1+-12+		4+ - 12+		
cumulation E = 1	N	%	Ν	%	
Stock	5289302	100	349185	100	
M	1948045	37	71533	20	
$S_j$	41766	1	36003	10	
SAV	3299491	62	241648	69	

intensity and protected fish sizes. The basic values in the model were  $N_0$  and parameters  $M_i$ ,  $F_i$ , and  $B_i$ 1 of the matrices. The distribution of the data remained the same as in Option 3, and only fishing intensity  $E_i$ was added to the matrix:  $F_i$ 1 =  $F_i \cdot E_i$ . Fishing intensity with nets E was increased subsequently every 0.3 for a fragment of matrix  $B_i$ , which referred to fish in second (1+), third (2+), and fourth (3+) years of life. Fishing intensity for the other age groups of 5, 6...13 remained at an unchanged level of E = 1. The operation of the model was thus verified, where in matrix various values of intensity were applied for the requirement of increasing fishing pressure on younger fish, while leaving unchanged the fishing pressure on older fish.

The following examples were simulated:

- description of the common bream population at initial fishing intensity with nets and rods at *E* = 1 based on materials and catches from the 1995-2001 period;
- 2. the highest fishing intensity *E* every 0.3 in ten steps and three variants: increased fishing intensity with nets and a constant rod fishing effort; increased fishing intensity with rods and a constant net fishing effort; increased fishing intensity with both fishing systems;
- 3. changes in the actual net effort that were applied in Zegrzyńskie Reservoir to a level that was applied at the time and with a constant intensity of rod fishing; for the requirements of this simulation, the net effort was transformed to the intensity of fishing with nets according to the formula  $E = q_{E=1} \cdot meff$  where

q – the conversion factor of the intensity of fishing with nets E = 1, was determined for the mean annual gill-net days for the 1997-2001 period of data collection, which was 0.000153374233; *meff* – mean annual gill-net days in subsequent long-term periods of fisheries catches in Zegrzyńskie Reservoir; for the 1997-2001 period it was 6520; the values determined for the index of fishing intensity served to perform the simulation of changes in the size of common bream catches with nets and rods in subsequent periods of the existence of the reservoir;

- changes in the sizes of fish caught and the fishing intensity with nets and searching for the effects of this simulation in changes in population abundance;
- 5. the variant with changes to the natural mortality parameter and the effects of this in stock survival and changes in catch size;
- 6. determining the economic values of the catches after changes in the actual fishing effort with nets throughout the existence of the reservoir.

## Results

# Population and the size of catches at a constant value of the fishing intensity index

Table 2 presents the parameters of the structure of the common bream population in Zegrzyńskie Reservoir:  $N_0$  is the initial number of spawn or fry, fish abundance  $N_i$  in *i*-*nth* period (i = 1, 2, 3, ...t), natural mortality  $M_i$ , catches of fish in units (individuals) and in kilograms with nets and rods  $F_{ij}$ . In the first and second years of life natural mortality  $M_i$  was the highest and affected more than 98% of the population. With the passage of time, two mortality parameters influenced the older year classes of the population: natural  $M_i$ , which stabilized, and fishing  $F_{ij}$ , which was low initially, but increased as the fish aged. At constant effort of net and rod catches described by fishing intensity index E = 1, mean annual catches of

#### Table 4

Common bream population after a four-fold increase in fishing intensity with nets and rods: fish abundance in the stock, natural mortality M, caught  $S_i$ , and those remaining in the reservoir  $S_{AV}$ 

Age group cumulation Intensity $E = 4$		1+ - 12+		4+ - 12+	4+ - 12+		
		N	%	Ν	%		
nets	Stock	5289302	100.0	349185	100.0		
	M	1948045	36.8	65591	18.8		
	$S_j$	58024	1.1	45282	<b>10.9</b> / 2.0		
	$S_{AV}$	3283233	62.1	238312	68.2		
rods	Stock	5289302	100.0	349185	100.0		
	M	1948045	36.8	69454	19.9		
	Sj	54326	1.0	46386	3.3 / <b>10.0</b>		
	$S_{AV}$	3286931	62.1	233344	66.8		
nets + rods	Stock	5289302	100.0	349185	100.0		
	M	1948045	36.8	63209	18.1		
	$S_j$	64607	1.2	48255	7.5 / 6.3		
	S <sub>AV</sub>	3276650	61.9	237721	68.1		

#### Table 5

Fishing effort with nets (gill-net days) with assigned values of fishing intensity E in the model

		Fishing effort v	Fishing effort with nets				
Years	Ν	gill-nets	days	$meff_x$	Ε		
1965-1981	7	91	163	14833	2.275		
1983-1990	5	65	163	10595	1.625		
1991-1995	4	52	163	8476	1.3		
1997-2001	3	40	163	6520	1		
2002-2004	2	26	163	4238	0.65		
2005-2006	1	13	163	2119	0.325		
2007-	1	13	114	1482	0.227		

common bream with nets and rods was 25,390 ind. and 19,491 kg and 16,376 ind. and 10,409 kg, respectively (calculated from Table 2).

During period *i*, the stock lost fish to natural mortality and fishing (Table 3). The abundance of the entire stock in the second year of life (1+) at the beginning of the season was 5,289,302 ind. Approximately 37% of the fish died of natural causes, which is 1,948,045 ind. Approximately 1% was reduced by commercial and recreational exploitation. The reservoir retained 3,299,491 fishes, which is approximately 62% of the stock. The stock that achieved the age of sexual maturity (4+ and older) numbered

349,185 fishes. Of this, 20% died naturally, which is 71,533 units, and commercial and rod catches took another 10%, which was 36,003 fishes, which left 69% of the mature fish.

## Population and catch size after changing the values of the fishing intensity index

Figures 2a-c present the relation of changes in catch size between the net and rod exploitation systems under the influence of changing fishing intensity E in three variants. Increasing the effort of net catches

#### Table 6

Changes in the abundance of fish in the population following simulated changes in the fishing system and legal size limits and increasing indicator of fishing intensity E

Fishing system		Simulation	S <sub>j</sub> .	$Y_{j}$ .	$w_{j}$ .	е
nets -	<i>E</i> = 1	>	25390	19491	0.77	2924849348
changes in system -	E'	>	86647	17518	0.20	761061684
increased E'-	1.3	>	93514	18390	0.20	541786612
	1.6	>	107056	19234	0.18	429670857
	1.9	>	119345	19920	0.17	339742232
	2.2	>	130649	20452	0.16	263578396
	2.5	>	141016	20829	0.15	203972041
	2.8	>	150646	21119	0.14	154097612
	3.1	>	159504	21317	0.13	116442201
	3.4	>	167608	21448	0.13	84407825
	3.7	>	175219	21535	0.12	62678735
	4.0	>	182520	21575	0.12	43842186

#### Table 7

Changes in population abundance and catches of common bream after the simulation with changes in the values of mortality parameter M

M		<i>M</i> (0+)	Stock(1+)	<i>M</i> (1+)		F		$S_{AV}$	
change	parameter	Ni	$N_i$	$N_i$	%	$S_{ij}$	%	$N_i$	%
M+0.06	0.2304987	2924656212	411611	193136	46.9	13489	3.28	204986	49.8
M+0.03	0.2138638	2923778757	2581213	1070591	41.5	25115	0.97	1485507	57.6
M+0.01	0.2038949	2923193788	4308005	1655560	38.4	36256	0.84	2616188	60.7
М	0.1990889	2922901303	5289302	1948045	36.8	41766	0.79	3299491	62.4
M-0.01	0.1943807	2922608818	6382000	2240530	35.1	40014	0.63	4101456	64.3
M-0.03	0.1852192	2922023848	8818647	2825500	32.0	39897	0.45	5953250	67.5
M-0.06	0.1720189	2921146393	13222371	3702955	28.0	39821	0.30	9479595	71.7

resulted in a gradual increase in the size of fish catches. Simultaneously, while maintaining a constant rod effort, the size of recreational catches decreased (Fig. 2a). The reverse possibility in the second variant presents the same scenario of change in catch sizes (Fig. 2b); increased rod effort resulted in increased recreational catches with a simultaneous decrease in catches with nets. Increasing the fishing intensity index with nets and rods simultaneously resulted in increased catches for both gear types. In all variants, increasing the size of catches was gradual and the function was curvilinear. Doubling the size of fish

catches required a quadruple increase in fishing intensity with nets or rods.

The spread between catches in units and kilograms of fish increased along with fishing intensity *E*, and consequently the unit weight of fish in the catches decreased (Figs. 2a-c). When the fishing effort with nets and rods was increased simultaneously, the mean unit weight decreased substantially (Fig. 2c). This was the direct result of increasing catches of smaller fish as the fishing effort increased.

In this variant, a four-fold increase in fishing intensity *E* with nets and rods in the stock of fish aged 1 + do



Figure 2. Catches and mean common bream unit weight after changing the intensity of fishing with nets *E*: at a constant intensity of fishing with rods (a); with rods at a constant intensity of fishing with nets (b); nets and rods combined (c).

12+, with 36.8% of the fish dying as a result of natural mortality, resulted in catching 1.1% of the fish. If we take the part of the stock into consideration that was in the fully catchable phase, which was ages 4+ to 12+, close to 18.8% of the fish died as a result of natural mortality, and net and recreational catches combined took 12.9%. This left close to 68.2% of the total number of fish, which was 238,312 ind. (Table 4).

Changes in rod fishing intensity produced similar simulation results. Of the portion of the stock that was in the fully catchable phase (ages 4+ and older), natural mortality resulted in the death of 19.9%, recreational catches and those with nets took 13.3%, while 66.8% remained in the reservoir, which was 233,344 fishes. Increasing fishing intensity with nets and rods simultaneously resulted in a catch size of 13,8%, with 18.1% of the fish succumbing to natural mortality leaving 68.1% of the fish, or 237,721 ind., in the reservoir (Table 4).

# Population and catch size after changes in actual fishing intensity with nets

The examples presented above refer to hypothetical changes in fishing intensity with nets and rods. The actual fishing effort with nets in Zegrzyńskie Reservoir decreased as the years passed (Table 5), and, consequently, the size of catches decreased. From a level of approximately 40,000 ind. and 26 tons of common bream at a fishing intensity with nets of E = 2.275 (14,833 gill-net days), catches with nets decreased to over 8,500 idv. and nearly 8 tons at an intensity of E = 0.227 (1,482 gill-net days). During this period, recreational catches of common bream

increased from 10,000 ind. and 6.6 tons by 1.7 and 2 times, respectively (Fig. 3).

The effects of changes in the actual fishing effort with nets was illustrated by the share in the stock of fish aged 4+ and older that succumbed to natural mortality  $M_i$ , were caught  $S_{jj}$ , and which remained in the reservoir  $S_{AV}$  (Fig. 4). The exploitation system that has been used to date has not had a significant impact on the common bream population. Despite changes in fishing effort with nets, a huge segment of this stock remains in the reservoir. At the highest intensity of fishing effort with nets of E = 2.275, 68.1% of the exploited stock remained in the reservoir, while at an intensity of E = 0.227, 71.7% remained (Fig. 4).



Figure 3. Simulation of changes in catch size (S – units; Y – kg) of common bream with nets *j*. and rods .*j* after reducing the actual fishing effort with nets in the 1965-2007 period.



Figure 4. Stock dynamics at age 4+ and older following the reduction in the actual intensity of fishing with nets  $E_{j}$ . in the 1965-2007 period in Zegrzyński Reservoir.

# Population and catches after introducing changes to the sizes of fish caught and reducing reproductive potential

The result of decreasing the size of fish caught after using seines was to increase catches substantially in units and to decrease the unit weight of the fish, which led to decreased overall catch weight (Table 6). The reproductive potential determined empirically at nearly 3 billion eggs was reduced to 761 million eggs by shifting fishing to catch younger fish. Increasing fishing intensity caused a further reduction in the stock and decreased reproductive potential. The catch increased decidedly in units; the weight increase was barely perceptible. In the last step, a four-fold increase in fishing intensity with nets resulted in increased catches to 182,000 ind. at a weight of just 21 tons. The reproductive potential was reduced to approximately 44 million eggs (Table 6).

## Population and catches after changing mortality parameter *M* at constant fishing intensity *E*

Table 7 presents examples of hypothetical variants of changes in parameter M at constant fishing intensity

E = 1 and the effects on the population and catches. The intensity of mortality in the first year of life M(0+) changed as a result of introducing changes to parameter M. Decreasing the natural mortality parameter in three steps - M-0.01; M-0.03; M-0.06 gradually reduced the mortality of fish 0+. Simultaneously, more fish survived to the first period of the second year of life (Stock1+). The number of fish dying M(1+) still increased, but their percentage share decreased. Decreasing mortality parameter M did not have a significant impact on catch size Sij. In each variant of the simulation this oscillated at about 40,000 units. When parameter M was increased by M +0.01, M +0.03, and M +0.06, respectively, the number of fish dying in the first year of life M(0+) increased. Fewer fish survived until the first period of age 1+ (Stock1+), and the abundance of fish still dying decreased, while their percentage share in reference to stock abundance at the beginning of the period increased. The abundance of the stock segment that remained in the reservoir  $S_{AV}$  decreased. Under increased mortality *M*, the size of the catches decreased in every step of the simulation up to 13,489 units in the last one.



Figure 5. Changes in the economic value of common bream caught with nets and rods with changes in the actual intensity of fishing with nets  $E_{i}$ .

# Economic value of catches when changing the magnitude of exploitation

Reducing the size of common bream net catches in subsequent periods must have resulted in decreased economic profits (Fig. 5). Beginning with the intensity of fishing with nets E = 1, profits collapsed. During the period analyzed, the economic value of these fish catches fish decreased 2.3 times. Simultaneously, during this period the economic value of recreational catches of common bream increased 2.26 times (Fig. 5).

## Discussion

The Numerical Model of Fish Exploitation is a simulation model designed for researching exploited fish populations. It was created with the aim of seeking possibilities for steering salmonid management, and following modifications, it was applied to simulating commercial catches of fish in lakes and dam reservoirs (Sych 1972, 1976, Sych and Wilkońska 1978, Sych and Bartel 1979, Nabiałek 1995). The current version of the model has been adjusted for conducting assessments of different fish populations (mentioned in Buras and Wiśniewolski 2011, 2014). Like other fisheries models (Kazanskiy et al. 1986, Shybaev 1986, Miller 2001, Hewitt et al. 2007, Kinzey and Punt 2009), the biological and exploitation data included in the composition of NMFE have a dichotomous, matrix structure.

The example of a common bream population was used to present the possible applications of the NMFE model to determine population size and structure, the size of catches with changes in catch effort, natural and fishing mortality, the sizes of fish caught, reproductive potential, and size types. The simulations illustrated population dynamics and forecasted changes in reaction to changes introduced in population and exploitation parameters (Kompowski and Horbowy 1990).

Similar structural simulations have been conducted with other models (Bandura and Shybaev 1986, Kazanskiy et al. 1986, Shybaev 1986), and the results of these simulations indicated that commercial and recreational catches depicted similar changes to those presented in this paper. This shows that a catch system based primarily on set nets and recreational catches did not influence a significant reduction in a common bream population. This result corresponds to the observations of other authors (Kazanskiy et al. 1986, Horpilla and Peltonen 1994).

Regulating high fish abundance and through this reducing biomass can be achieved by changing the size structure of catches or by changing the intensity and system of exploitation (EIFAC-FAO 1991, Horppila and Peltonen 1994, Lammens et al. 2004). Such an effect can obtained, for example, by shifting fishing pressure onto younger year classes (2 + and 3 +), as a result of which fish populations can be reduced by as much as 70% (Biró 1978) and increased fishing mortality of young year classes can prevent further fish biomass growth (Helminen et al. 1993, Horppila and Peltonen 1994). This study obtained the hypothetical effect of regulating fish abundance by simulating changes in the size structure of the catches to younger fish. The problem, however, extends beyond the practical aspects of such solutions since the model simulates changes in fishing mortality parameter F. New catches that are designated hypothetically mean increasing the effort of fishing with nets and rods, but very high F with a shift to younger fish can mean it is necessary to introduce another fishing system.

At a constant fishing intensity and with constant biotic and abiotic conditions, the stock is constantly supplemented with new generations (Beverton and Holt 1957, Tyurin 1962, Biró 1978). In this work, natural mortality *M* was assumed to be a permanent, stable parameter of the common bream population determined based on catch, initial abundance of eggs treated as the reproductive potential of the entire population, and with the hypothesis that the highest M values occur in the first year of life and then decrease gradually to stabilize in older fish (Nabiałek 1995). As a result of reasons of chance in the environment, natural mortality can change and result in increased fish mortality or, conversely, in exceptionally high fish survival (Regier 1962, Biró 1978, Pauly 1980). In this model, the designated constant M parameter was changed to follow the effects on population size and catch size. Simulating changes in population abundance and catch size included the hypothesis of fish mortality under the influence of changing biological and environmental reasons of chance (Pauly 1980). Reducing natural fish mortality in the population did not impact the size of catches;

conversely, increases resulted in decreases in the size of this catch.

Parameter M was treated holistically as the sum of all natural causes of fish death: density, predation, aging, abiotic factors, and other causes (Pauly 1980, Hewitt et al. 2007). Direct methods for calculating this parameter require the regular tagging of large numbers of fish, a high intensity of recapturing these fish, or directly counting the fish in a population from a closed area, and they are extremely expensive (Wiśniewolski 1992). Thus, indirect methods are sought that assume there is a strict correlation between the biological parameters of a population and its abundance in different periods of life and catch sizes and total mortality (Marten 1978, Clark 1991, Paloheimo and Chen 1996, Abella et al. 1997, Hewitt and Hoenig 2005, Hewitt et al. 2007, Kinzey and Punt 2009, Kienzle 2016). Determining population parameters is basically the first fundamental step in the further analysis of fish populations (Alagaraja 1984, Gonçalves et al. 2003, García and Duarte 2006, Powers 2014, Gaertner 2015).

NMFE is a single-species model, and studies conducted with it refer to the reaction of one population to exploitation. Multi-species models are recommended for commercial catches since they are more sensitive, and they analyze parameter M as a component of various natural causes, especially the relationships between predators and prey. These models can also be applied successfully to fisheries that are smaller and local and also to recreational catches (Rechency et al. 2017). Recommendations in favor of multi-species models stem from these models' higher sensitivity to changes in population parameters, their more realistic approach to these parameters, and a smaller error load in simulation results (Abella et al. 1997, Quinn and Collie 2005, Kinzey and Punt 2009, Powers 2014). Some researchers have pointed out differences in estimated population parameters stemming from the estimated size of the fished stock, the portion of spawning stock, or additional stock in the results obtained between single-species and multi-species models (Kinzey and Punt 2009, Gislason 1999, Jurado-Molina et al. 2016). On the other hand, single-species models are appropriate especially when fish populations are maintained for catches (Rechencq et al. 2017). Single-species models are still a basic tool used for assessing the state of fisheries resources and for managing them (Quinn and Collie 2005, Powers 2014).

Verifying that the model operates properly with regard to the actual state is exceedingly problematic. This stems from the very nature of the model, which describes the virtual reality of the population. Sometimes comparisons are undertaken between simulation results and actual catches to verify conformity (Mandecki 1976, Majkowski 1977, Kazanskiy et al. 1986). In reference to simulating catch sizes based on fishing effort with nets, which were applied in subsequent periods of fisheries management in the Zegrzyńskie Reservoir, the results of NMFE simulations of the size of mean annual catches with nets in the 2005-2008 period were close to the actual catches made with nets, which were 7.9 tons per year according to catch statistics from this period. For earlier periods, the hypothetical mean annual catches expressed in weight that were calculated with the model differed from the actual mean annual catches of common bream. This discrepancy could have resulted from the simulation using the current mean fish weight in the catches, when in the 1970s and 1980s the mean weight of the fish caught was higher. At an intensity of E = 2.275 for the 1964-1981 period, the simulated catches were 39,000 ind, and 25 tons, while the actual figure was 39 tons. However, if the mean unit weight of 1 kg from this period had been used in the model, then the weight value would have been approximately 40 tons. The model depends on weights, but the final result aims to determine the abundance of fish caught.

From the point of view of the fisheries, three basic elements comprise the exploitation of fish resources: the size of catches expressed in weight, the mean weight of fish in the catches, population reproduction measured as the number of eggs (Bandura and Shybaev 1986). The first two elements characterize the effectiveness of the fisheries and the size and level of commercial catches. The third element characterizes the possibilities of rebuilding populations under exploitation conditions. A fourth element can be added, which is the economic scope of exploitation expressed as money. The results of model simulations indicated that increased effort did not cause proportional increases in catch sizes. The increasing spread between the size of catches in units and the weight of these catches indicated that as effort was increased, the unit weight of fish decreased. Increased catches of smaller common bream, the price for which was lower, resulted in decreases in the overall economic value of the catches (Kompowski and Horbowy 1990).

High demand for the exploitation of inland water fish resources has led to the development of refined methods to assess the size of exploited population of different species. The model presented in this paper was a useful tool for assessing the state of populations of different fish species (Sych 1976, Sych and Wilkońska 1978, Nabiałek 1995, Buras and Wiśniewolski 2011). However, further work should strive to expand the structure and functioning of the model with the aim of increasing its capabilities, e.g., to assess the state and exploitation of multi-species assemblages.

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